# Sample Assignment 6
## Discussed on 2020-10-26,
*Not graded*

**Question 1 (Minimum and Maximum Heap Invariant.**

**(A)** Assume that heap is implemented as a 0-based array (the root element is $H[0]$), and the heap supports DELETEMIN(H) operation that should remove the minimum element (and return the heap into consistent state).

(*Note.* A consistent state in such a heap means that the key in parent does not exceed keys in left and right child.)

Find, if the heap property holds in the following array:

$$H[0] = 6, 17, 25, 20, 15, 26, 30, 22, 33, 31, 20.$$

If it is not satisfied, find, which two keys you could swap in this array so that the heap property is satisfied again. Write the correct sequence of array $H$.

**(B)** Assume that heap is implemented as a 0-based array (the root element is $H[0]$), and the heap supports DELETEMAX(H) operation that should remove the maximum element.

If the heap does not satisfy invariant (in a consistent max-heap, every parent should always be at least as big as both children), then show how to swap two nodes to make it correct.

$$96, 67, 94, 10, 67, 68, 69, 9, 10, 11, 50, 67.$$

**Question 2 (Insert into a Min-Heap.**

Show what is the final state of a heap after you insert number 6 into the following "minimum-heap" (represented as a zero-based array):

$$9, 18, 28, 23, 20, 29, 33, 25, 36, 34, 23.$$

**Question 3 (Delete maximum from a Max-Heap.**

Show what is the final state of a heap after you remove the maximum from the following heap (represented as a zero-based array):

$$96, 67, 94, 10, 67, 68, 69, 9, 10, 11, 50, 67.$$

**Question 1. (A)** Answer:
$6, 15, 25, 20, 17, 26, 30, 22, 33, 31, 20.$
To see, which numbers we need to swap, we draw the original array as a tree (and verify the heap invariant).
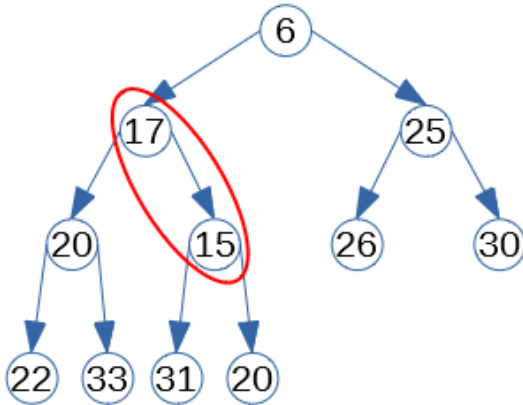


Figure 1: Violated min-heap invariant.

**(B)** Answer:
$96, 67, 94, 10, 67, 68, 69, 9, 10, 11, 50, 67.$
This tree is already correct (regarding the max-heap property).