Sample Assignment 8

Not graded

Introduction.
Encode General Tree to a Binary One

Sometimes non-binary (ordered) trees should be represented in binary-tree data structures. See `https://bit.ly/3khnC0p` for details. The two rules to encode are these:

- Every node v in the general tree with its **first child** w maps to the same node v in the binary tree, where the corresponding w is its **left child**.
- Every node v in the general tree having w as its **sibling to the right** has the same w in the binary tree as its **right child**.

One can also decode: given a binary tree (if its root only has the left child), it is possible to restore the original general tree.
Consider an example general tree on Figure 1.

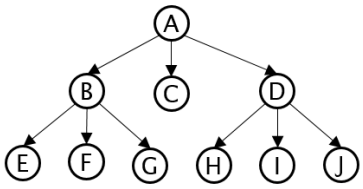

Figure 1: The given general tree.

Encoding Step 1 Redraw edges (only connect each node with its first child and also to the sibling to the right). To see clearly which edges will be left-going, and which are right-going, can color them differently. See Figure 2.

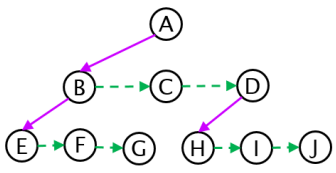

Figure 2: Reordering the levels.

Encoding Step 2 Adjust the levels in the new binary tree so that it takes a more conventional look (left children to the left, right children to the right). See Figure 3.

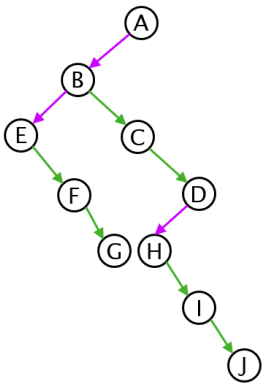

Figure 3: Binary tree with edges colored.

Question 1: Decode to a General Tree

(A) List all the nodes in Figure 4 using the in-order tree traversal.

(B) Binary tree B shown in Figure 4 has been obtained by encoding some general tree T (The original tree T was rooted and ordered, but it is not necessarily binary.)
Restore the general tree T by decoding the given binary tree.

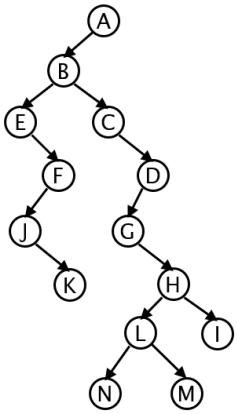

Figure 4: Binary tree to Convert to a General Tree

Question 1:
(A)

$$E, J, K, F, B, C, G, N, L, M, H, I, D, A.$$

(B)
Note. In your answer there is no need to draw the intermediate decoding steps; they are just for your information.

Decoding Step 1. Can color left and right edges as in Figure 5.

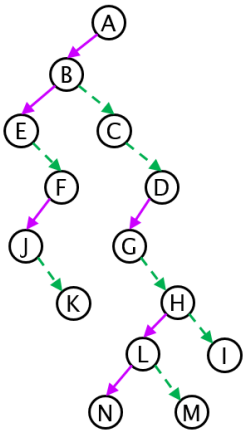

Figure 5: Colored binary tree

Decoding Step 2. Now look at the previous Figure 5, but bend your head to the right (so that the green arrows seem horizontal).
We can rearrange levels so that the right-going edges stay on the same level (but left-going edges go one level down). See Figure 6.

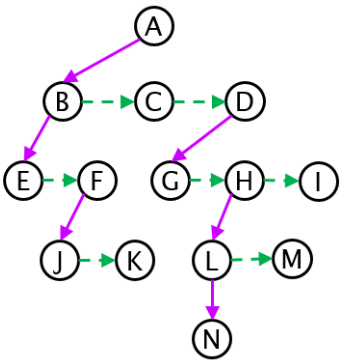

Figure 6: Reordered by Level

Decoding Step 3 (Final Answer). Once the layers are restored, we can forget about the binary tree edges (the violet and green ones), but instead connect the parent with all the children (i.e. not just the first child, but also all its siblings to the right). Now the original tree is restored. See Figure 7.

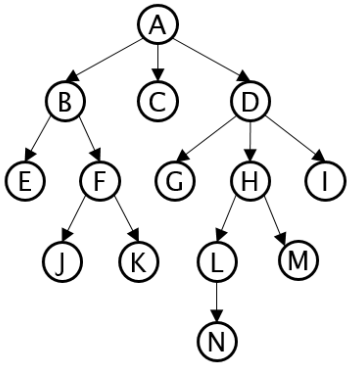

Figure 7: Final Answer