Worksheet 12: Trees

Question 1. Count the objects:
(A) If T is a tree with 999 vertices, then T has ... edges.
(B) There are ... non-isomorphic trees with four vertices.
(C) There are ... non-isomorphic rooted trees with four vertices (isomorphism for rooted trees can change map any node to any other; but it should map root to root).
(D) There are ... full binary trees with six vertices.
(E) The cycle graph C_7 has ... spanning trees.
(F) If T is a binary tree with 100 vertices, its minimum height is ....
(G) If T is a full binary tree with 101 vertices, its minimum height is ....
(H) If T is a full binary tree with 101 vertices, its maximum height is ....
(I) If T is a full binary tree with 50 leaves, its minimum height is ....
(J) Every full binary tree with 61 vertices has ... leaves.
(K) Every full binary tree with 50 leaves has ... vertices.
(L) Every 3-ary tree with 13 vertices has ... leaves.

Question 2. Find, if a statement is true or false:
(A) If T is a tree with 17 vertices, then there is a simple path in T of length 17.
(B) Every tree is bipartite.
(C) There is a tree with degrees $3, 2, 2, 2, 1, 1, 1, 1, 1$.
(D) There is a tree with degrees $3, 3, 2, 2, 1, 1, 1, 1$.
(E) If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.
(F) If T is a tree with 50 vertices, the largest degree that any vertex can have is 49.
(G) In a binary tree with 16 vertices, there must be a path of length 4.
(H) If T is a rooted binary tree of height 5, then T has at most 25 leaves.

Question 3.
Suppose you have 50 coins, one of which is counterfeit (either heavier or lighter than the others). You use a balance scale to find the bad coin. Prove that 4 weighings are not enough to guarantee that you find the bad coin and determine whether it is heavier or lighter than the other coins.

Question 4.
Suppose you have 5 coins, one of which is counterfeit (either heavier or lighter than the other four). You use a pan balance scale to find the bad coin and determine whether it is heavier or lighter.
(A) Prove that 2 weighings are not enough to guarantee that you find the bad coin and determine whether it is heavier or lighter.
(B) Draw a decision tree for weighing the coins to determine the bad coin (and whether it is heavier or lighter) in the minimum number of weighings.

Question 5.
Suppose you have 5 coins, one of which is heavier than the other four. Draw the decision tree for using a balance scale to find the heavy coin. How many weighings would you need?

Question 6.

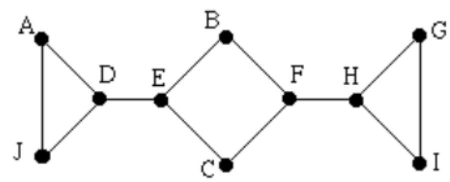

Figure 1. Graph for DFS and BFS traversal.

(A) Using alphabetical ordering, find a spanning tree for the graph on Figure 1 by using a depth-first search.
(B) Using alphabetical ordering, find a spanning tree for this graph by using a breadth-first search.
(C) Using the ordering $C, D, E, F, G, H, I, J, A, B$, find a spanning tree for this graph by using a depth-first search.
(D) Using the ordering $C, D, E, F, G, H, I, J, A, B$, find a spanning tree for this graph by using a breadth-first search.
(E) Using reverse alphabetical ordering, find a spanning tree for the graph by using a depth-first search.
(F) Using reverse alphabetical ordering, find a spanning tree for the graph by using a breadth-first search.

Question 7.
Write the compound proposition $(\neg p) \rightarrow (q \vee (r \wedge \neg s))$ as the abstract syntax tree ($\neg$, $\rightarrow$, $\vee$ and $\wedge$ operators are inner nodes; but p, q, r, s are leaves).
List the graph nodes in pre-order, in-order and post-order traversal of this syntax tree.

Question 8.
Draw the abstract syntax tree, the preorder and postorder traversal of $(8x - y)^5 - 7\sqrt{4z - 3}$.

Question 9. The string
$p\ r\ q\ \rightarrow\ \neg\ q\ \triangle\ p\ \rightarrow\ \wedge$
is postfix notation for a logic expression; however, there is a misprint. The triangle should be one of these three: r, $\vee$, or $\neg$. Determine which of these three it must be and explain your reasoning.

Answers

Question 1. Answer:
(A) If T is a tree with 999 vertices, then T has 998 edges.
(B) There are 2 non-isomorphic trees with four vertices.
(C) There are 4 non-isomorphic rooted trees with four vertices.
(D) There are 0 full binary trees with six vertices.
(E) The cycle graph C_7 has 7 spanning trees.
(F) If T is a binary tree with 100 vertices, its minimum height is 6.
(G) If T is a full binary tree with 101 vertices, its minimum height is 6.
(H) If T is a full binary tree with 101 vertices, its maximum height is 50.
(I) If T is a full binary tree with 50 leaves, its minimum height is 6.
(J) Every full binary tree with 61 vertices has 31 leaves.
(K) Every full binary tree with 50 leaves has 99 vertices.
(L) Every 3-ary tree with 13 vertices has 9 leaves.

Question 2. Answer: TBD

Question 3. Answer:

Four weighings could give at most $3^4 = 81$ outcomes (each time we use scales there are three possible results). But the problem has $2 \cdot 50$ different situations (any of the 50 coins can be different; and it can be either heavier or lighter).

Question 4. Answer: TBD

Question 5. Answer: TBD

Question 6. Answer: TBD

Question 7. Answer: TBD

Question 8. Answer: TBD

Question 9. Answer: TBD