Final Review: Counting

Discrete Structures

(Final scheduled for Wednesday, April 28, 2021)

You must justify all your answers to recieve full credit

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5 Counting

Divisibility, GCM and LCM, congruences, multiplicative inverses, exponentiation.

5.(a). Given a word problem, map bijectively to a set expression (including unions, Cartesian products, etc.) and count variants using the product, sum, difference rules.

5.(b). Given a set of restrictions and symmetries inherent for the task, count variants using the division rule.

5.(c). Given a selection task, use combination and permutation formulas with or without repetition.

5.(d). Given a polynomial, find coefficients using binomial and multinomial rules.

5.(e). Given a word problem, estimate the "worst case" using the pigeonhole principle.

5.(f). Given a word problem, count variants using inclusion-exclusion principle.

5.(a). Count the number of parenthesized expressions using 4 variable names (A, B, C, D), three unary minuses ($-$) and up to 4 unary minuses ($\sim$). (Two parenthesized expressions are considered different iff they have different syntax trees – they lead to a different order of expression evaluation.)

5.(b). Combinatorial problems involving symmetry:

(a) Somebody has 5 pennies and 4 jars. All pennies are distinguishable and each penny should go into some jar. All the jars are placed in the vertices of a square $ABCD$; jar locations that differ only by rotations of that square are considered indistinguishable. (For example, if all the pennies from the vertex A would go to B; all pennies from B would go to C, from C to D, and from D to A, then it would be the same way to distribute pennies.) Find the number of ways how the pennies can be distributed among these jars. Write your answer as an integer.

(b) A *poker hand* is an (unordered) collection of any 5 different cards (out of the standard set of 52 playing cards). A poker hand is named "two pair" if it contains two cards of one rank, two cards of another rank and one more card. For example, J♥, J♣, 4♣, 4♦, 9♥. Find the total number of all poker hands that are "two pair".

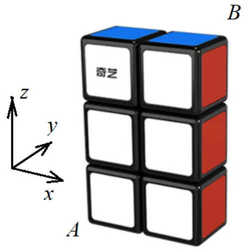
5.(c). There are altogether 11 subjects in some school and each student must register for exactly 4 of them. How many students are needed so that at least 2 students are registered for exactly the same subjects?

5.(d). For each question find the integer number:

(a) In how many ways can the letters in word `PEPPER` be rearranged?

(b) Consider the expression that occurs if $(a + b + c)^6$ is expanded (all parentheses opened). What is the coefficent forr the term ab^3c^2?

(c) Assume that an ant wants to travel from A to B in the parallelepiped with dimensions $1 \times 2 \times 3$. It has to travel two units along x axis, one unit along y axis and three units along z axis (Figure 5.(d).b). How many paths do there exist? (Ant can travel also inside this shape, but it has to travel one full unit edge of a cube before it can change its direction.



5.(e). For each property find the minimum number of people that must be in a group to ensure that the property holds (or specify that such minimum does not exist, since the property may fail for arbitrarily large groups of people):

(a) At least 2 people were born on the same day of the year. (Keep just the date; ignore the year of birth. Assume there are 365 days in a year; ignore leap years.)

(b) At least 2 people were born on two adjacent dates (adjacent dates are April 28 and April 29 and also December 31 and January 1).

(c) At least 2 people were born on January 1st.

(d) At least 3 people were born on the same day of the week. (Assume there are 7 days in a week.)

(e) At least 4 people were born in the same month.

(f) At least one month is such that the number of people born during that month exceeds the number of days in that month by 5 or more.

5.(f). Count the number of functions $f : A \to A$ from the set $A = \{1,2,3,4,5,6,7,8,9\}$ to itself such that the three inverse images $f^{-1}(1)$, $f^{-1}(2)$, $f^{-1}(3)$ are all nonempty.