Final Review: Graphs

Discrete Structures
(Final scheduled for Wednesday, April 28, 2021)

You must justify all your answers to recieve full credit

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1 Graphs

Keywords: Vertex, edge, (induced) subgraph, (un)directed, simple, regular, bipartite, Handshaking theorem, connectedness, path, cycle, Euler circuit, complement, isomorphisms.

1.1 Topics

1.(a). Given a special graph (complete, cycle, wheel, n-cube, complete bipartite), check some property of the graph.

1.(b). Given a graph, check if it is bipartite, complete, or connected.

1.(c). Justify whether or not a graph has a particular subgraph (such as a path, cycle, complete graph, etc)

1.(d). Convert between different representations of graphs (pair of sets, diagram, adjacency list, adjacency matrix).

1.(e). Given two graphs prove or disprove they are isomorphic.

1.(f). Given a property of a graph (edge / vertex number, regularity), estimate the number of graphs with that parameter.

1.(g). Given a graph, check the condition for a Euler circuit (or path) and find it, if it exists.

1.2 Sample questions

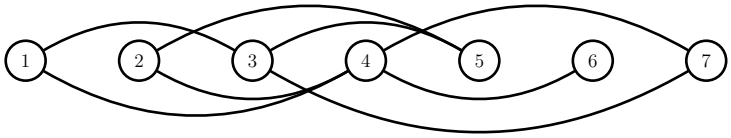
1.(a). Let G be an undirected graph.

i. If G has k edges and is connected, what is the smallest and largest possible number of vertices that G could have?

ii. If G is 4-regular and has n vertices, how many edges must it have?

1.(b). i. If G is bipartite, with vertex partitions of size 6 and 10, what is the smallest and largest number of edges that G could have?

ii. Show that the graph below is bipartite.

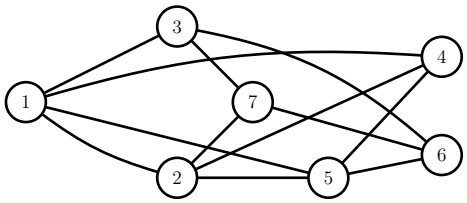


1.(c). In the following graph, find:

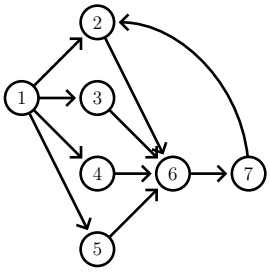
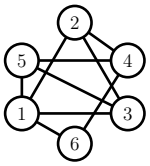
i. K_4 as an induced subgraph

ii. a 7-cycle

iii. as many different 3-cycles as possible

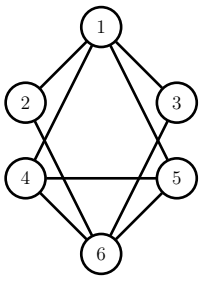


1.(d). Complete the table below, where each column is a graph and each row is a way to represent the graph. The first column is a directed graph, and the rest are undirected.

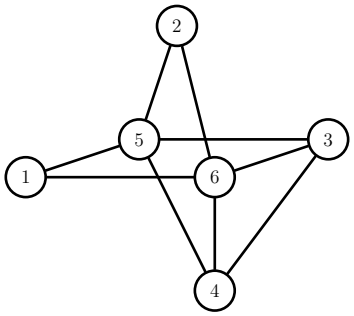
graph drawing				
adjacency matrix			$\begin{bmatrix} 0&0&0&1&1&1 \\ 0&0&0&1&1&1 \\ 0&0&0&1&1&1 \\ 1&1&1&0&0&0 \\ 1&1&1&0&0&0 \\ 1&1&1&0&0&0 \end{bmatrix}$	
incidence matrix				$\begin{bmatrix} 1&0&0&0&0&0&0&1 \\ 0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&1&1&0 \\ 0&0&0&0&1&1&0&0 \\ 1&1&0&0&0&0&0&0 \\ 0&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1 \\ 0&0&0&1&1&0&0&0 \end{bmatrix}$

1.(e). Describe an explicit isomorphism between the two graph below. That is, define a bijection $V_G \to V_H$ between the two vertex sets and show that it induces a bijection $E_G \to E_H$.

G :



H :



1.(f). Let G be a simple 2-regular graph with 9 vertices.

i. If G is undirected, how many non-isomorphic choices are there for G?

ii. If G is directed, how many non-isomorphic choices are there for G?

1.(g). Construct a graph with 10 vertices and 20 edges that has an Euler circuit, and one that does not.

2 Trees

Keywords: Leaf, root, height, n-ary tree, balanced tree, traverse trees, BFS, DFS.

2.1 Topics

2.(a). Given some parameters of a rooted tree (count of vertices, internal vertices, leaves, edges, height), estimate other parameters.

2.(b). Given a (full and/or balanced) n-ary tree and some parameters, estimate other parameters.

2.(c). Convert between different representations of a tree (diagram, syntax tree, prefix / infix / postfix notation, preordered / inordered / postordered vertices with children)

2.(d). Given an undirected graph, do a DFS and BFS traversal to find a spanning tree, indicating all steps.

2.2 Sample questions

2.(a). Let T be a tree with 20 vertices. What is the smallest / largest possible number of internal vertices, leaves, edges that T could have?

2.(b). For each of the following rooted trees identify:

i. for which n the tree is n-ary,

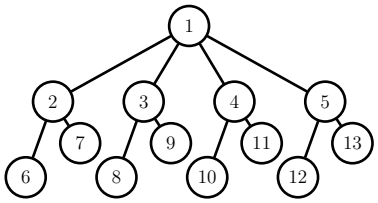
ii. whether or not it is full,

iii. whether or not it is balanced,

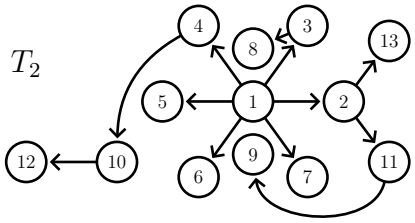
iv. how many internal nodes it has,

v. how many leaves it has.

T_1

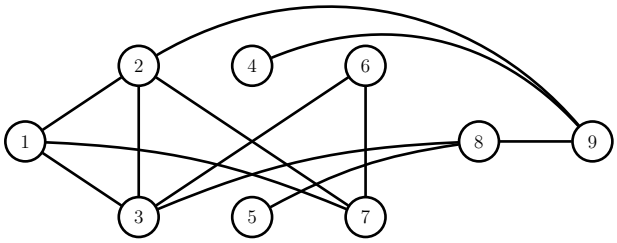


T_2



2.(c). Convert the following preordered vertex list into a diagram, an inordered vertex list, and a postordered vertex list: $abcnghfedokjlim$. The leaves are n, g, f, d, o, l, m.

2.(d). Let G be the following undirected graph.



i. Indicate all the steps of DFS starting at vertex 1 and at vertex 9.

ii. Indicate all the steps of BFS starting at vertex 1 and at vertex 9.