Midterm Review

Discrete Structures

You must justify all your answers to recieve full credit

Please refer to the following resources to find question samples:
Midterm (24.02.2021) Questions only: `https://bit.ly/3t638wK`
Midterm (24.02.2021) Solved: `https://bit.ly/2Oy4cu6`
Question Samples: `https://bit.ly/3t5p4YO`

1. **Boolean expressions.** Truth tables, logical equivalences, set operations, Venn diagrams.
   (a) Given a statement in English and atomic propositions, write its Boolean expression. (Midterm, Q1)
   (b) Given a Boolean expression, fill in missing values in its truth table.
   (c) Given a Boolean expression equivalently transform it using Boolean identities.
   (d) Given a Boolean expression, prove or disprove a tautology.
   (e) Given a truth table, create a DNF or a CNF for it (and vice versa).
   (f) Given a set expression, shade the regions in a Venn diagram that belong to it. (Midterm, Q2)
   (g) Given two set expressions prove or disprove set identity or subset relation. (Midterm, Q3)
2. **Quantifiers.** Predicates, quantifiers, precedence, simple proofs.
   (a) Given an English sentence and predicates, write its predicate expression.
   (b) Given a predicate expression, restore parentheses, identify free/bound variables.
   (c) Given a predicate expression, write its negation (De Morgan laws etc.). (Midterm, Q4)
   (d) Given truth tables for predicates, evaluate nested quantifier expressions. (Question Samples, 2(d))
   (e) Given a description of a set, define it in a set-builder notation. (Midterm, Q6)
   (f) Given a pseudocode, write the predicate expression that it computes.
3. **Functions.** Injections, surjections, bijections,
   (a) Given a function in curly bracket notation, determine its values and its range.
   (b) Given a function, prove/disprove that it is injective, surjective or bijective. (Midterm, Q7)
   (c) Given function definitions, evaluate their compositions and inverses.
   (d) Given a sequence, identify its properties, is it (eventually) constant/periodic, etc.
   (e) Given an expression with elementary functions, $|x|$, $\lfloor x \rfloor$, $\lceil x \rceil$, evaluate it.
   (f) Given an expression $\sum_{i=0}^{n} \ldots$ and similar constructs, evaluate it.
   (g) Given an expression with 2×2, 2×1 matrices, add, subtract and multiply them.
4. **Sets of Numbers.** Properties of integers, rationals/irrationals, reals.
   (a) Given an expression, write a forward proof that it is rational or irrational.
   (b) Given an expression, write a proof by contradiction that it is irrational. (Midterm, Q8)
   (c) Given an expression, give examples to disprove that it must be rational/irrational.
   (d) Given a set, describe its powerset and estimate cardinality (by Cantor's theorem). (Midterm, Q9)

(e) Given functions, use them to compare cardinalities (also Schröder-Bernstein theorem).

5. **Big-O notation.**

   (a) Given functions f, g, check by definition that $f(n)$ is in $O(g(n))$, $\Omega(g(n))$, $\Theta(g(n))$.

   (b) Given a function $f(x)$, simplify it to get its "optimal" $O(g(x))$ or $\Theta(g(x))$ class. (Midterm, Q10)

   (c) Given a collection of functions, arrange them by growth.

   (d) Given a pseudocode, basic operations and input length, estimate its time as $O(g(n))$.

   (e) Given a sorting algorithm and input data, trace its action on this data.

6. **Number Theory.** Congruences. Bezout identity. Inverses. Chinese remainder theorem.

   (a) Given a number, factorize it as a product of primes and prime powers.

   (b) Given a number n and its divisor d, divide with remainder as $n = qd + r$.

   (c) Given an arithmetic progression, find its kth member $\pmod m$.

   (d) Given two integers, find their GCD (also LCM) by Euclid algorithm. (Midterm, Q11)

   (e) Given integers, solve Bezout identity (or find inverses) with Blankenship algorithm.

   (f) Given a in integer power, simplify it using Little Fermat theorem.

   (g) Given a system of congruences, solve it using Chinese remainder theorem. (Midterm, Q12)

7. **Numbers in different bases.** Binary, octal, decimal, hexadecimal.

   (a) Given a decimal integer, convert it to binary, hexadecimal (and vice versa).

   (b) Given a binary integer, convert it into octal and hexadecimal (and vice versa).

   (c) Given a number in one base, estimate its length in another base.

   (d) Given two binary numbers, add or multiply them with the school algorithm. (Midterm, Q13)

   (e) Given a periodic fraction, write it as P/Q (use infinite geometric progression). (Midterm, Q14)

   (f) Given numbers a, b, m, estimate fast exponentiation result (and time used) for $a^b \pmod m$.

   (g) Given a fraction $p/2^k$, convert it into finite binary/hexadecimal fractions.

8. **Induction.** Mathematical induction, strong induction. Recursive definitions.

   (a) Given an equality regarding sums or recurrent sequences, prove it by induction. (Midterm, Q15)

   (b) Given an inequality, prove it by induction.

   (c) Given a parametrized statement (using parameter n), prove it by strong induction.

   (d) Given a recurrent definition of a sequence $f(n)$, evaluate it for some value n.

   (e) Given a coin weighing, search or similar task, find the recurrence for its time.